## UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Advanced Level

## MATHEMATICS

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

1 Packets of fish food have weights that are distributed with standard deviation 2.3 g . A random sample of 200 packets is taken. The mean weight of this sample is found to be 99.2 g . Calculate a $99 \%$ confidence interval for the population mean weight.

2 A mathematics module is assessed by an examination and by coursework. The examination makes up $75 \%$ of the total assessment and the coursework makes up $25 \%$. Examination marks, $X$, are distributed with mean 53.2 and standard deviation 9.3. Coursework marks, $Y$, are distributed with mean 78.0 and standard deviation 5.1. Examination marks and coursework marks are independent. Find the mean and standard deviation of the combined mark $0.75 X+0.25 Y$.

3 Random samples of size 120 are taken from the distribution $B(15,0.4)$.
(i) Describe fully the distribution of the sample mean.
(ii) Find the probability that the mean of a random sample of size 120 is greater than 6.1 .

4 A certain make of washing machine has a wash-time with mean 56.9 minutes and standard deviation 4.8 minutes. A certain make of tumble dryer has a drying-time with mean 61.1 minutes and standard deviation 6.3 minutes. Both times are normally distributed and are independent of each other. Find the probability that a randomly chosen wash-time differs by more than 3 minutes from a randomly chosen drying-time.

5 The random variable $X$ has probability density function given by

$$
\mathrm{f}(x)= \begin{cases}4 x^{k} & 0 \leqslant x \leqslant 1 \\ 0 & \text { otherwise }\end{cases}
$$

where $k$ is a positive constant.
(i) Show that $k=3$.
(ii) Show that the mean of $X$ is 0.8 and find the variance of $X$.
(iii) Find the upper quartile of $X$.
(iv) Find the interquartile range of $X$.

6 A dressmaker makes dresses for Easifit Fashions. Each dress requires $2.5 \mathrm{~m}^{2}$ of material. Faults occur randomly in the material at an average rate of 4.8 per $20 \mathrm{~m}^{2}$.
(i) Find the probability that a randomly chosen dress contains at least 2 faults.

Each dress has a belt attached to it to make an outfit. Independently of faults in the material, the probability that a belt is faulty is 0.03 . Find the probability that, in an outfit,
(ii) neither the dress nor its belt is faulty,
(iii) the dress has at least one fault and its belt is faulty.

The dressmaker attaches 300 randomly chosen belts to 300 randomly chosen dresses. An outfit in which the dress has at least one fault and its belt is faulty is rejected.
(iv) Use a suitable approximation to find the probability that fewer than 3 outfits are rejected.

7 The number of cars caught speeding on a certain length of motorway is 7.2 per day, on average. Speed cameras are introduced and the results shown in the following table are those from a random selection of 40 days after this.

| Number of cars caught speeding | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of days | 5 | 7 | 8 | 10 | 5 | 2 | 3 |

(i) Calculate unbiased estimates of the population mean and variance of the number of cars per day caught speeding after the speed cameras were introduced.
(ii) Taking the null hypothesis $\mathrm{H}_{0}$ to be $\mu=7.2$, test at the $5 \%$ level whether there is evidence that the introduction of speed cameras has resulted in a reduction in the number of cars caught speeding.
(iii) State what is meant by a Type I error in words relating to the context of the test in part (ii). Without further calculation, illustrate on a suitable diagram the region representing the probability of this Type I error.

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